

Logical Modelling of Physarum Polycephalum

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Abstract. In the paper we proposed a novel model of unconventional computing where a structural part of computation is presented by dynamics of Plasmodium of Physarum polycephalum, a large single cell. We sketch a new logical approach combining conventional logic with process calculus to demonstrate how to employ formal methods in design of unconventional computing media presented by Physarum polycephalum.

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1 Introduction

In the paper we are demonstrating how to design unconventional computing media taking into account the problem that in unconventional computing media both structural parts and computing data of computers are variable ones, in particular in reaction-diffusion processors [11, 1] we are dealing with, both the data and the results of the computation are encoded as concentration profiles of the reagents (on the contrary, in conventional models

of computation one makes a distinction between the structural part of a computer, which is fixed, and the data which are variable and on which the computer operates). As a result, while in conventional models circuits are fixed, in unconventional computing media circuits could be set up just as dynamically variable ones within spatio-temporal logic. Solving this task allows us to build up nature-inspired computer models and to consider biological and physical systems as computational models.

One of the unconventional, nature inspired models similar to reaction-diffusion computing is chemical machine in that molecules are viewed as computational processes supplemented with a minimal reaction kinetics. Berry and Boudol first built up a chemical abstract machine [44, 4] as an example of how a chemical paradigm of the interactions between molecules can be utilized in concurrent computations (in algebraic process calculi). We are considering another abstract machine of reaction-diffusion computing exemplified by dynamics of *Plasmodium* of *Physarum polycephalum*. This machine is constructed by using process algebra, too. Using it, we are studying a possibility of logical representation of the computation in reaction-diffusion systems. Using notions of space-time trajectories of local domains of a reaction-diffusion medium we will could the spatial logic of trajectories, where well-formed formulas and their truth-values are defined in the unconventional way

Experimental studies and designs of reaction-diffusion computers could be traced back to the pioneer discovery of L. Kuhnert (1986). He demonstrated that some very basic image transformations can be implemented in the light-sensitive Belousov-Zhabotinsky system. The ideas by L. Kuhnert., K. L. Agladze, I. Krinsky (1989) on image and planar shape transformations in two-dimensional excitable chemical media were further developed and modified by N. G. Rambidi (1998). At that time, during the mid and late nineties, a range of chemical logical gates were experimentally built in the Showalter and Yoshikawa laboratories. The first chemical laboratory prototypes of precipitating chemical processors for computational geometry were developed by A. Adamatzky (1996). He also designed and studied a range of hexagonal cellular-automaton models of reaction-diffusion excitable chemical systems. Now reaction-diffusion computing is an extremely

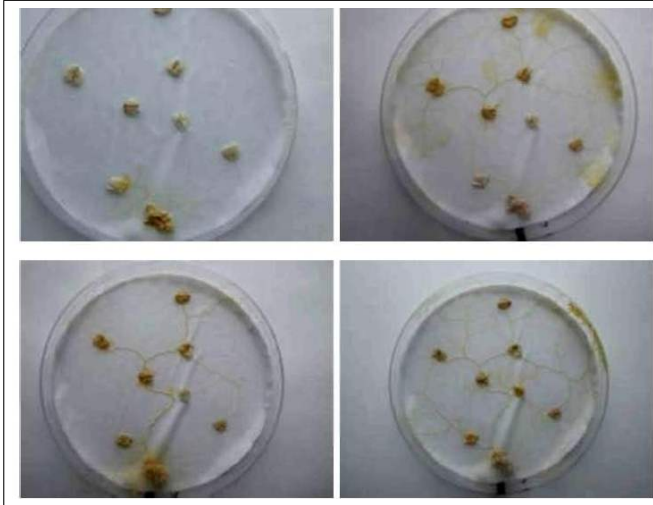


Figure 1: An example of computational process in Physarum machine. Photographs are taken with time lapse circa 24 hours.

wide area of researches towards unconventional computing.

The dynamics of plasmodium of *Physarum polycephalum* could be regarded as one of the natural reaction-diffusion computers. The point is that when the plasmodium is cultivated on a nutrient-rich substrate (agar gel containing crushed oat flakes) it exhibits uniform circular growth similar to the excitation waves in the excitable Belousov-Zhabotinsky medium (Fig. 1). If the growth substrate lacks nutrients, e.g. the plasmodium is cultivated on a non-nutrient and repellent containing gel, a wet filter paper or even glass surface localizations emerge and branching patterns become clearly visible (Fig. 2, 3).

The plasmodium continues its spreading, reconfiguration and development till there are enough nutrients. When the supply of nutrients is over, the plasmodium either switches to fructification state (if level of illumination is high enough), when sporangia are produced, or forms sclerotium (encap-

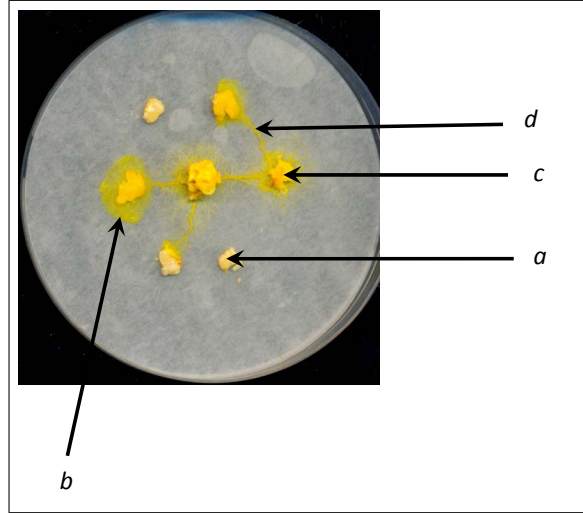


Figure 2: Basic components of Physarum and its environment: (a) oat flake, (b) propagating pseudopodium, plasmodium's wave-fragment, (c) oat flake colonized by plasmodium, (d) protoplasmic tube.

sulates itself in hard membrane), if in darkness.

The pseudopodium propagates in a manner analogous to the formation of wave-fragments in sub-excitable Belousov-Zhabotinsky systems. Starting in the initial conditions the plasmodium exhibits foraging behavior, searching for sources of nutrients (Fig. 1). When such sources are located and taken over, the plasmodium forms characteristic veins of protoplasm, which contracts periodically. Belousov-Zhabotinsky reaction and plasmodium are light-sensitive, which gives us the means to program them. Physarum exhibits articulated negative phototaxis, Belousov-Zhabotinsky reaction is inhibited by light. Therefore by using masks of illumination one can control dynamics of localizations in these media: change a signal's trajectory or even stop a signal's propagation, amplify the signal, generate trains of signals. Light-sensitive of Plasmodium has been already explored in design of

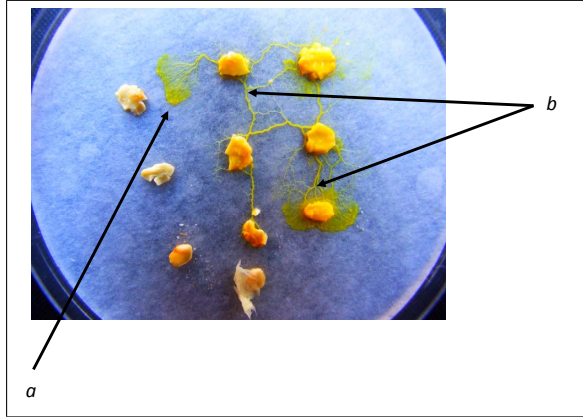


Figure 3: Snapshot of experimental dish with propagating plasmodium, new activated propagating zone (a) and sites of branching pseudopodia, junctions of protoplasmic tubes (b) are shown by arrows.

robotics controllers [22, 2], [33, 3].

Despite numerous experimental implementations of Physarum computers there is a lack of formalization of the plasmodium's behavior abstract enough to infer high-level principle of information transmission by the plasmodium and accurate enough to reflect peculiarities of the Physarum foraging behavior. In the paper we are trying to fill the gap and offer interpretation of Physarum behavior in a framework of process calculi. In the paper we are trying to define Physarum machine as a process calculus built on an unconventional interpretation of logical connectives.

2 A logical approach to analyzing Physarum machine

Let us set up the problem how to define logical connectives in Physarum machine. The matter is that both structural parts and computing data of

Physarum computers are variable.

Physarum machine may be viewed as a labelled transition system, which consists of a collection of states $\mathcal{L} = \{p_{ij}\}_{i=\overline{1,N}, j=\overline{1,K}}$ and a collection \mathcal{T} of transitions (processes, actions) over them. Assume $\mathcal{T}: \mathcal{L} \mapsto \mathcal{P}(\mathcal{L})$, where $\mathcal{P}(\mathcal{L}) = \{T: T \subseteq \mathcal{L}\}$. This means that $\mathcal{T}(p)$ consists of all states that a reachable from p . The transition system is understood as a triple

$$\langle \mathcal{L}, \mathcal{T}, \longrightarrow \rangle,$$

where $\longrightarrow \subseteq \mathcal{L} \times \mathcal{T} \times \mathcal{L}$ is a transition relation that models how a state $p \in \mathcal{L}$ can evolve into another state $p' \in \mathcal{L}$ due to an interaction $\sigma \in \mathcal{T}$. Usually, $\langle p, \sigma, p' \rangle \in \longrightarrow$ is denoted by $p \xrightarrow{\sigma} p'$. So, a state p' is reachable from a state p if $p \xrightarrow{\sigma} p'$.

The finite word $\alpha_1\alpha_2\ldots\alpha_n$ is a *finite trace of transition system* whenever there is a finite execution fragment of transition system

$$\varrho = p_0\alpha_1p_1\alpha_2\ldots\alpha_np_n \text{ such that } p_i \xrightarrow{\alpha_{i+1}} p_{i+1} \text{ for all } 0 \leq i < n.$$

The word $\alpha_1\alpha_2\ldots\alpha_n$ is denoted by $\text{trace}(\varrho)$. The infinite word $\alpha_1\alpha_2\ldots$ is an infinite trace whenever there is an infinite execution fragment of transition system

$$\varrho = p_0\alpha_1p_1\alpha_2p_2\alpha_3p_3\ldots \text{ such that } p_i \xrightarrow{\alpha_{i+1}} p_{i+1} \text{ for all } 0 \leq i.$$

The word $\alpha_1\alpha_2\ldots$ is denoted by $\text{trace}(\varrho)$ too.

Definition 2.1. *An infinite (resp. finite) trace of state p denoted by $\varrho(p)$ is the trace of an infinite (resp. finite) execution fragment starting in p .*

Each trace can be regarded as a graph, where nodes represent states and edges transitions. In this way, transition system is viewed as graph trees. Conventionally, logical connectives are defined in the algebraic way that is broken within transition systems. Therefore in Physarum machine we will distinguish two kinds of logical connectives:

1. *logical connectives defined co-algebraically*, these ones are closed to conventional logical connectives: they are “eternal,” because they are defined over traces (i.e. for any future states);

2. *logical connectives defined as transitions over states*, these ones differ from conventional logical connectives: they are defined over states, therefore their values could change for some future states.

2.1 Logical connectives defined co-algebraically

First, let us consider logical connectives defined co-algebraically, i.e. by coinduction.

An infinite trace of state $\varrho(p)$ may be presented as a kind of stream. For a trace $\varrho(p)$, we call $\varrho_p(0)$ the initial value of $\varrho(p)$. We define the *derivative* $\varrho_p(0)$ of a trace $\varrho(p)$, for all $n \geq 0$, by $\varrho'_p(n) = \varrho_p(n+1)$. For any $n \geq 0$, $\varrho_p(n)$ is called the n -th state of $\varrho(p)$. It can also be expressed in terms of higher-order trajectory derivatives, defined, for all $k \geq 0$, by $\varrho_p^{(0)} = \varrho(p)$; $\varrho_p^{(k+1)} = (\varrho_p^{(k)})'$. In this case the n -th state of a trace $\varrho(p)$ is given by $\varrho_p(n) = \varrho_p^{(n)}(0)$. So, the trajectory is understood as an infinite sequence of derivatives: $\varrho(p) = \varrho_p(0) :: \varrho_p(1) :: \varrho_p(2) :: \dots :: \varrho_p(n-1) :: \varrho_p^{(n)}$, or $\varrho(p) = \langle \varrho_p(0), \varrho_p(1), \varrho_p(2), \dots \rangle$.

A *bisimulation* on the set of traces is a relation R such that, for all $\varrho(p)$ and $\varrho(q)$, if $\langle \varrho(p), \varrho(q) \rangle \in R$ then (i) $\varrho_p(0) = \varrho_q(0)$ (this means that they have the same *initial value*) and (ii) $\langle \varrho'_p, \varrho'_q \rangle \in R$ (this means that they have the same *differential equation*).

If there *exists* a bisimulation relation R with $\langle \varrho_p, \varrho_q \rangle \in R$ then we write $\varrho_p \sim \varrho_q$ and say that ϱ_p and ϱ_q are *bisimilar*. In other words, the *bisimilarity* relation \sim is the greatest bisimulation. In addition, the bisimilarity relation is an equivalence relation.

Theorem 2.1 (Coinduction). *For all $\varrho(p), \varrho(q)$, if there exists a bisimulation relation R with $\langle \varrho(p), \varrho(q) \rangle \in R$, then $\varrho(p) = \varrho(q)$.* \square

This proof principle is called *coinduction*. It is a systematic way of proving the statement using bisimilarity: instead of proving only the single identity $\varrho(p) = \varrho(q)$, one computes the greatest bisimulation relation R that contains the pair $\langle \varrho(p), \varrho(q) \rangle$. By coinduction, it follows that $\varrho(p) = \varrho(q)$ for all pairs $\langle \varrho(p), \varrho(q) \rangle \in R$.

Now consider logical connectives defined by coinduction over traces. Their syntax is as follows:

Variables: $\mathbf{p} ::= p \mid q \mid r \dots,$

where p, q, r are states of Physarum machine presented as a labelled transition system.

Constants: $\mathbf{c} ::= \top \mid \perp$

where \top means the truth (the ideal, universal trace) and \perp means the falsity (the empty, impossible trace).

Formulas: $\varphi, \psi ::= \mathbf{p} \mid \mathbf{c} \mid \neg\psi \mid \varphi \vee \psi \mid \varphi \wedge \psi \mid \varphi \supset \psi$

These definitions are coinductive. For instance,

- a variable \mathbf{p} is of the form of a trace $\mathbf{p} = \mathbf{p}(0) :: \mathbf{p}(1) :: \mathbf{p}(2) :: \dots :: \mathbf{p}(n-1) :: \mathbf{p}^{(n)}$, where $\mathbf{p}(i) \in \{p, q, r, \dots\}$ for each $i \in \omega$;
- a constant \mathbf{c} is of the form of a trace $\mathbf{c} = \mathbf{c}(0) :: \mathbf{c}(1) :: \mathbf{c}(2) :: \dots :: \mathbf{c}(n-1) :: \mathbf{c}^{(n)}$, where $\mathbf{c}(i) \in \{\top, \perp\}$ for each $i \in \omega$, a particular case is $[\top] = [\top](0) :: [\top](1) :: [\top](2) :: \dots :: [\top]^{(n)}$, where $[\top](i) = \top$ for each $i \in \omega$;
- a formula $\neg\varphi$ has the differential equation $(\neg\varphi)' = \neg(\varphi')$ and its initial value is $(\neg\varphi)(0) = \neg\varphi(0)$, this formula will be understood as $\varphi \supset [\perp]$;
- a formula $\varphi \vee \psi$ has the differential equation $(\varphi \vee \psi)' = \varphi' \vee \psi'$ and its initial value is $(\varphi \vee \psi)(0) = \varphi(0) \vee \psi(0)$;
- a formula $\varphi \wedge \psi$ has the differential equation $(\varphi \wedge \psi)' = \varphi' \wedge \psi'$ and its initial value is $(\varphi \wedge \psi)(0) = \varphi(0) \wedge \psi(0)$;
- a formula $\varphi \supset \psi$ has the differential equation $(\varphi \supset \psi)' = \varphi' \supset \psi'$ and its initial value is $(\varphi \supset \psi)(0) = \varphi(0) \supset \psi(0)$.

2.2 Logical connectives defined as transitions over states

In analyzing the plasmodium we observe processes of inaction, fusion, and choice, which could be interpreted as unconventional (spatial) falsity, conjunction and disjunction respectively, denoted by Nil , $\&$ and $+$. These operations differ from conventional ones, because they cannot have a denotational semantics in the standard way. However, they may be described as special transitions over states of Physarum machine:

1. inaction (Nil) means that pseudopodia has just stopped to behave,
2. fusion ($\&$) means that two pseudopodia come in contact one with another and then merge,
3. choice ($+$) means a competition between two pseudopodia in their behaviours.

Let us notice that a Boolean algebra may be extended up to the case of the system of logical connectives defined by coinduction [55, 5] (see the previous subsection). However, if we define three basic logical connectives (falsity, conjunction, disjunction) as transitions over states of Physarum machine, they will be extremely non-classical and Boolean properties do not hold for them in general case.

3 Physarum process calculus

Further, let us try to build up a process calculus combining two approaches to logical connectives for describing the dynamics of Physarum machine, i.e. we are trying to show that indeed this machine could be presented as a labelled transition system with some logical relations.

Assume that the computational domain Ω is partitioned into computational cells $c_j = 1, \dots, K$ such that $c_i \cap c_j = \emptyset$, $i \neq j$ and $\bigcup_{j=1}^K c_j = \Omega$. Then suppose that in the K cells, there are N active species or growing pseudopodia and the state of species i in cell j is denoted by p_{ij} , $i = 1, \dots, N$,

$j = 1, \dots, K$. These states are time dependent and they are changed by plasmodium's active zones interacting with each other and affected by attractants or repellents. Plasmodium's active zones interact concurrently and in a parallel manner. Foraging plasmodium can be represented as a set of following abstract entities (Fig. 2).

1. The set of *active zones* (growing pseudopodia or actions) $Z = \{a, b, \dots\}$ (Fig. 2a). On a nutrient-rich substrate plasmodium propagates as a typical circular, target, wave, while on the nutrient-poor substrates localized wave-fragments are formed. Each action a from Z belongs to a state p_{ij} , $i = 1, \dots, N$, $j = 1, \dots, K$ of a cell i , which is its current position, and says about a transition (propagation) of a state p_{ij} to another state of the same or another cell. Part of plasmodium feeding on a source of nutrients may not propagate, so its transition is nil, but this part can always start moving.
2. The set of *attractants* $\{A_1, A_2, \dots\}$ are sources of nutrients, on which the plasmodium feeds. It is still subject of discussion how exactly plasmodium feels presence of attracts, indeed diffusion of some kind is involved. Based on our previous experiments we can assume that if the whole experimental area is about $8 - 10 \sim cm$ in diameter then the plasmodium can locate and colonize nearby sources of nutrients. Each attract $A(a)$ is a function from a to another action b .
3. The set of *repellents* $\{R_1, R_2, \dots\}$. Plasmodium of *Physarum* avoids light. Thus, domains of high illumination are repellents such that each repellent R is characterized by its position and intensity of illumination, or force of repelling. In other words, each repellent $R(a)$ is a function from a to another action b .
4. The set of *protoplasmic tubes* $\{C_1, C_2, \dots\}$. Typically plasmodium spans sources of nutrients with protoplasmic tubes/veins (Fig. 2). The plasmodium builds a planar graph, where nodes are sources of nutrients, e.g. oat flakes, and edges are protoplasmic tubes. $C(a)$ means a diffusion of growing pseudopodia by an action a .

Our process calculus contains the following basic operators: ‘*Nil*’ (inaction), ‘ \star ’ (prefix), ‘ $|$ ’ (cooperation), ‘ \backslash ’ (hiding), ‘ $\&$ ’ (reaction/fusion), ‘ $+$ ’ (choice), a (constant or restriction to a stable state), $A(\cdot)$ (attraction), $R(\cdot)$ (repelling), $C(\cdot)$ (spreading/diffusion). Let $\Lambda = \{a, b, \dots\}$ be a set of names. With every $a \in \Lambda$ we associate a complementary action \bar{a} . Define $L = \{a, \bar{a} : a \in \Lambda\}$, where a is considered as *activator* and as *inhibitor* for a , be the set of labels built on Λ (under this interpretation, $a = \bar{\bar{a}}$). Suppose that an action a communicates with its complement \bar{a} to produce the internal action τ . Define $L_\tau = L \cup \{\tau\}$.

We use the symbols α, β , etc., to range over labels (actions), with $a = \bar{\bar{a}}$, and the symbols P, Q , etc., to range over processes on states p_{ij} , $i = 1, \dots, N$, $j = 1, \dots, K$. The processes are given by the syntax:

$$P, Q ::= Nil \mid \alpha \star P \mid A(\alpha) \star P \mid R(\alpha) \star P \mid C(\alpha) \mid (P|Q) \mid P \backslash Q \mid P \& Q \mid P + Q \mid a$$

Each label is a process, but not vice versa. An operational semantics for this syntax is defined as follows:

$$\begin{aligned} \textbf{Prefix:} \quad & \frac{}{\alpha \star P \xrightarrow{\alpha} P}, \\ & \frac{}{A(\alpha) \star P \xrightarrow{\beta} P} \quad (A(\alpha) = \beta), \\ & \frac{}{R(\alpha) \star P \xrightarrow{\beta} P} \quad (R(\alpha) = \beta), \end{aligned}$$

(the conclusion states that the process of the form $\alpha \star P$ (resp. $A(\alpha) \star P$ or $R(\alpha) \star P$) may engage in α (resp. $A(\alpha)$ or $R(\alpha)$) and thereafter they behave like P ; in the presentations of behaviors as trees, $\alpha \star P$ (resp. $A(\alpha) \star P$ or $R(\alpha) \star P$) is understood as an edge with two nodes: α (resp. $A(\alpha)$ or $R(\alpha)$) and the first action of P),

$$\textbf{Diffusion:} \quad \frac{P \xrightarrow{\alpha} P'}{P \xrightarrow{\alpha} C(\alpha)} \quad (C(\alpha) ::= P'),$$

$$\mathbf{Constant:} \quad \frac{P \xrightarrow{\alpha} P'}{a \xrightarrow{\alpha} P'} \quad (a ::= P, a \in L_\tau),$$

$$\mathbf{Choice:} \quad \frac{P \xrightarrow{\alpha} P'}{P + Q \xrightarrow{\alpha} P'}, \quad \frac{Q \xrightarrow{\alpha} Q'}{P + Q \xrightarrow{\alpha} Q'},$$

(these both rules state that a system of the form $P + Q$ saves the transitions of its subsystems P and Q),

$$\mathbf{Cooperation:} \quad \frac{P \xrightarrow{\alpha} P'}{P|Q \xrightarrow{\alpha} P'|Q}, \quad \frac{Q \xrightarrow{\alpha} Q'}{P|Q \xrightarrow{\alpha} P|Q'},$$

(according to these rules, the cooperation — interleaves the transitions of its subsystems),

$$\frac{P \xrightarrow{\alpha} P' \quad Q \xrightarrow{\bar{\alpha}} Q'}{P|Q \xrightarrow{\tau} P'|Q'},$$

(i.e. subsystems may synchronize in the internal action τ on complementary actions α and $\bar{\alpha}$),

$$\mathbf{Hiding:} \quad \frac{P \xrightarrow{\alpha} P'}{P \setminus Q \xrightarrow{\alpha} P' \setminus Q} \quad (\alpha \notin Q, Q \subseteq L),$$

(this rule allows actions not mentioned in Q to be performed by $P \setminus Q$),

$$\mathbf{Fusion:} \quad \frac{}{\alpha \star P \& \bar{P} \xrightarrow{\alpha} Nil}$$

(the fusion of complementary processes are to be performed into the inaction),

$$\frac{P \xrightarrow{\alpha} P' \quad Q \xrightarrow{\alpha} P'}{P \& Q \xrightarrow{\alpha} P'}, \quad \frac{P \xrightarrow{\alpha} P' \quad Q \xrightarrow{\alpha} P'}{Q \& P \xrightarrow{\alpha} P'}$$

(this means that if we obtain the same result P' that is produced by the same action α and evaluates from two different processes P and Q , then P' may be obtained by that action α started from the fusion $P \& Q$ or $Q \& P$),

$$\frac{P \xrightarrow{\alpha} P'}{P \& Q \xrightarrow{\alpha} Nil + C(\alpha) + P'}, \quad \frac{P \xrightarrow{\alpha} P'}{Q \& P \xrightarrow{\alpha} Nil + C(\alpha) + P'}$$

(these rules state that if the result P' is produced by the action α from the processes P , then a fusion $P \& Q$ (or $Q \& P$) is transformed by that same α either into the inaction or diffusion or process P').

These are inference rules for basic operations. The ternary relation $P \xrightarrow{\alpha} P'$ means that the initial action P is capable of engaging in action α and then behaving like P' .

The informal meanings of basic operations are as follows:

1. Nil , this is the empty process which does nothing. In other words, Nil represents the component which is not capable of performing any activities: a deadlocked component.
2. $\alpha \star P$, a process $\alpha \in L$ followed by the process P : P becomes active only after the action α has been performed. An activator $\alpha \in L$ followed by the process P is interpreted as branching pseudopodia into two or more pseudopodia, when the site of branching represents newly formed process $\alpha \star P$.
In turn, an inhibitor $\bar{\alpha} \in L$ followed by the process P is annihilating protoplasmic strands forming a process at their intersection.
3. $A(\alpha) \star P$ denotes a process that waits for a value α and then continues as P . This means that an attractor A modifies propagation vector of action α towards P . Attractants are sources of nutrients. When such a source is colonized by plasmodium the nutrients are exhausted and attracts ceases to function: $A(\alpha) \star Nil$.
4. $R(\alpha) \star P$ denotes a process that waits for a value α and then continues as P . This means that a repellent R modifies propagation vector of action α towards P . Process can be cancelled, or annihilated, by a repellent: $R(\alpha) \star Nil$. This happens when propagating localized pseudopodium α enters the domain of repellent, e.g. illuminated domain, and α does not have a chance to divert or split.

5. $C(\alpha)$, a diffusion of activator $\alpha \in L$ is observed in placing sources of nutrients nearby the protoplasmic tubes belonging to α or inactive zone ($\alpha ::= Nil$). More precisely, diffusion generates propagating processes which establish a protoplasm vein (the case of activator α) or annihilate it (when source of nutrients exhausted, the case of inhibitor $\bar{\alpha}$).
6. $P|Q$, this is a parallel composition (commutative and associative) of actions: P and Q are performed in parallel. The parallel composition may appear in the case, two more food sources are added to either side of the array and then the plasmodium sends two streams outwards to engulf the sources. When the food sources have been engulfed, the plasmodium shifts in position by redistributing its component parts to cover the area created by the addition of the two new processes P and Q that will already behave in parallel.

Process P can be split, or multiplied, by two sources of attractants $(A_1A_2)(P) \star P_1|P_2$. Pseudopodium P approaches the site where distance to A_1 is the same as distance to A_2 . Then P subdivides itself onto two pseudopodia P_1 and P_2 . Each of the pseudopodia travels to its unique source of attractants. Also, process P can be split, or multiplied, by a repellent: $R(P) \star P_1|P_2$. Biophysics of fission with illuminated geometrical shapes is discussed in [5]. The fission happens when a propagating pseudopodium ‘hits’ a repellent. The part of pseudopodium most affected by the repellent ceases propagating, while two distant parts continue their development. Thus, two separate pseudopodia are formed.

7. $P \backslash Q$, this restriction operator allows us to force some of P ’s actions not to occur; all of the actions in the set $Q \subseteq L$ are prohibited, i.e. the component $P \backslash Q$ behaves as P except that any activities of types within the set Q are hidden, meaning that their type is not visible outside the component upon completion.
8. $P \& Q$, this is the fusion of P and Q ; $P \& Q$ represents a system which may behave as both component P and Q . For instance, Nil behaves

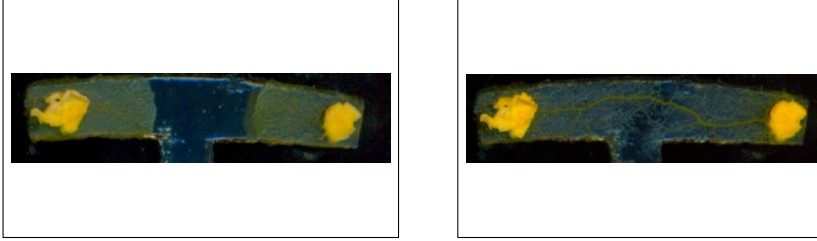


Figure 4: Merging of two plasmodium's wave-fronts. Photos are made with interval 9 hours.

as $P\&\bar{P}$, where P is an activator and \bar{P} an appropriate inhibitor respectively. The fusion of P and Q is understood as collision of two active zones P and Q . When they collide they fuse and annihilate, $P\&Q\star Nil$. Depending on the particular circumstances the new active zone α (the result of fusing) may become inactive (Nil), transform to protoplasmic tubes ($C(\alpha)$), or remain active and continue propagation in a new direction (the case of prefix \star).

When two pseudopodia come in contact one with another, they do usually merge (Fig. 4). Thus by directing processes with attractors we can merge the processes: $A(P_1, P_2) \star P_1\&P_2$ (see details in [5]).

9. $P + Q$, this is the choice between P and Q ; $P + Q$ represents a system which may behave either as component P or as Q . Thus the first activity to complete identifies one of the components which is selected as the component that continues to evolve; the other component is discarded. In Physarum calculi, the choice $P + Q$ between processes P and Q sometimes is represented by competition between pseudopodia tubes $C(P)$ and $C(Q)$, i.e. $C(P, Q) = C(P) + C(Q)$. In other words, two processes P and Q can compete with each, during this competition one process 'pulls' protoplasm from another process, thus making this another process inactive. The competition happens via protoplasmic tube.

10. a , constants belonging to labels are components whose meaning is given by equations such as $a ::= P$. Here the constant a is given the behaviour of the component P . Constants can be used to describe infinite behaviours, via mutually recursive defining equations.

Thus, in this process calculus we have two kinds of logical connectives.

1. The group of connectives defined by coinduction. They are derivable from the hiding. Indeed, let 1 be a universal set of active zones, then the following equalities hold:

$$\neg P ::= 1 \setminus P \quad \textbf{negation},$$

$$P \wedge Q ::= P \setminus (1 \setminus Q) \quad \textbf{conjunction},$$

$$P \vee Q ::= 1 \setminus ((1 \setminus P) \setminus Q) \quad \textbf{disjunction},$$

$$P \supset Q ::= 1 \setminus (P \setminus Q) \quad \textbf{implication}.$$

These connectives satisfy all properties of Boolean algebra.

2. The group of connectives defined as transitions. It consists of three operations: **inaction**, **fusion** and **choice**. Their basic properties:

$$Nil \setminus P \cong Nil, \quad (3.1)$$

$$P \& \bar{P} \cong Nil, \quad (3.2)$$

$$P \& P \cong P, \quad (3.3)$$

$$P \& Nil \cong Nil, \quad (3.4)$$

$$(P + Q) \setminus P' \cong P \setminus P' + Q \setminus P', \quad (3.5)$$

$$(P \& Q) \setminus P' \cong P \setminus P' \& Q \setminus P', \quad (3.6)$$

$$P \& Q \cong Q \& P, \quad (3.7)$$

$$P \& (Q \& R) \cong (P \& Q) \& R, \quad (3.8)$$

$$P + P \cong P, \quad (3.9)$$

$$P + Nil \cong P, \quad (3.10)$$

$$P + Q \cong Q + P, \quad (3.11)$$

$$P + (Q + R) \cong (P + Q) + R, \quad (3.12)$$

$$P \& (Q + R) \cong (P \& Q) + (P \& R), \quad (3.13)$$

$$P + (Q \& R) \cong (P + Q) \& (P + R), \quad (3.14)$$

where \cong is a congruence relation defined on the set of processes.

4 Conclusion

In the paper we have just shown that the behavior of plasmodium of Physarum polycephalum could be considered as a kind of process calculus with several logical connectives defined in non-standard way. Thus, the media of Physarum polycephalum can be viewed as one of the natural unconventional (reaction-diffusion) computers. Its weakest point is that *the speed of computation is so slow*: each new state of Physarum dynamics may be observed just in hours (see Fig. 1).

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